Radiative Properties of Solitons in an Assembly of Coupled Quantum Dots

Vijendra Kumar*, O.P. Swami, A.K. Nagar

Abstract— In this paper, we consider two dimensional assembly of coupled quantum dots interacting with the driving electromagnetic field. Keeping in view the local field effects, two discrete nonlinear Schrodinger equations are used to model the quantum dot (QD) assembly. We observe and study the interaction of solitons with quantum dots in the form of Rabi oscillations in QD assembly, coupled to electromagnetic field. Numerical calculations are performed and Rabi oscillations are analysed with the help of matlab software. As a result, we obtain properties and pattern of the radiative fields of the assembly in the modes of Rabi oscillations.

Index Terms — Discrete nonlinear Schrodinger equation, Electromagnetic Field, Local Field Effect, Matlab Software, Quantum Dot, Rabi Oscillations, Radiative Fields.

1 Introduction

The electronic properties of a quantum dot are closely related to its size and shape which allows the excitation and emission of quantum dots to be highly tunable[1]. The size of a quantum dot can be set during its formation to control its conductive propertives. Using many different sizes of quantum dots, we can achieve some quantum dot assemblies such as gradient multilayer nanofilms to exhibit a range of desirable emission properties. Therefore, the study of solitons in the assemblies of quantum dots has become an important topic of research.

One salient outcome of this study is that the radiative properties of solitons in the assembly of coupled quantum dots depend critically on the lattice geometry and symmetry. While on the other hand, the fundamental property of all type of solitons greatly depends on dimensionality. In this paper we consider a 2-dimensional assembly of coupled quantum dots interacting with the driving electromagnetic field. Taking into account the local field effects, two discrete nonlinear Schrodinger wave equations are used to model the assembly of coupled quantum dots. Local field effects also have their important role in the formation of excitonic Rabi oscillations in the self assembly of quantum dots [2, 3, 9 and 11]. In the form of Rabi oscillations, we observe the interaction between solitons and the assembly of quantum dots coupled to external electromagnetic field. Model equations are solved and Rabi oscillations are analysed with the help of matlab software. As a result, we obtain properties and pattern of the radiative fields of the assembly in the modes of Rabi oscillations.

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2 Basic Equations

Let us consider a two dimensional assembly of identical quantum dots with spacing dXd and it is exposed to the travelling wave in x-y plane with electric field

$$E(x, y, t) = \mathrm{Re}\{E_0 \exp[ik(x\cos\theta' + y\sin\theta') - i\omega t]\} \quad (1)$$

Here k is the wavenumber and θ' being the propagation angle. Quantum dots are taken as two level non-dissipative systems. The transition energy between excited electron orbital $|a_{m,n}|$ and ground state electron orbital $|b_{m,n}|$ is $\hbar\omega_0$. Here m & n denotes the position of quantum dots in x & y directions respectively. To ensure only intraband transitions, each quantum dot is coupled to four nearest quantum dots through electron tunneling in the assembly [4]. The interaction between light and quantum dots takes place in the strong regime so that the detuning frequency is small enough in comparison to both the optical as well as quantum transition frequencies [5, 6].

Let us consider a quantum dot for which rasing, lowering and population operators are denoted as:

$$\hat{\sigma}^{+}_{m,n} = |a_{m,n} >< b_{m,n}|,\ \hat{\sigma}^{-}_{m,n} = |b_{m,n} >< a_{m,n}|$$

And $$\hat{\sigma}_{z\,m,n} = |a_{m,n} >< a_{m,n}| - |b_{m,n} >< b_{m,n}| \quad (2)$$

The total Hamiltonian for this quantum dot can be written as –

$$\hat{H} = \hat{H}_d + \hat{H}_{df} + \hat{H}_T + \Delta\hat{H} \quad (3)$$

Here the term $\hat{H}_d$ describes the free electron motion

$$\hat{H}_d = \frac{\hbar\omega_0}{2}\sum_{m,n}\hat{\sigma}^{-}_{z\,m,n} \quad (4)$$

The term $\hat{H}_{df}$ corresponds to the interaction between quantum dots and electromagnetic field.

$$\hat{H}_{df} = -(\mu E_0)\sum_{m,n}\sigma^{+}_{m,n}\exp(i(m\phi_1 + n\phi_2)) + H.c. \quad (5)$$

H.c. stands for Hermitian conjugate operator, the phases

$\phi_1 = (\frac{kd}{2})\cos(\theta)$ and $\phi_2 = (\frac{kd}{2})\sin(\theta)$ represent the field delay per lattice period due to finite propagation speed. Here θ is the spherical coordinate. The term $\hat{H}_T$ in equation (3) describes the part of Hamiltonian for quantum dot with interdot coupling through tunneling.

The last term $\Delta\hat{H}$ of equation (3) corresponds to the local fields which can be modeled by Hartee-Fock-Bogoliubov approximations [7, 8] as follows:

$$\Delta\hat{H} = \frac{4\pi}{V} M_{\alpha,\beta}\mu_\alpha\mu_\beta \sum_{m,n}\left(\hat{\sigma}^-_{m,n}\left\langle\hat{\sigma}^+_{m,n}\right\rangle + \hat{\sigma}^+_{m,n}\left\langle\hat{\sigma}^-_{m,n}\right\rangle\right) \qquad (6)$$

Here $M_{\alpha,\beta}$ is the depolarization tensor of a quantum dot while μ_α & μ_β are components of the dipolar moment vector. V denotes to the volume of a single quantum dot. Now the temporal evolution of single-particle excitations can be governed by the Schrodinger equation

$$i\hbar\frac{\partial\left|\Psi\right\rangle}{\partial t} = \hat{H}\left|\Psi\right\rangle \qquad (7)$$

Let us take a wavefunction $\Psi(t)$ in the form of a coherent superposition as:

$$\left|\Psi(t)\right\rangle = \sum_{m,n}\left(\psi_{m,n}(t)e^{(\frac{i}{2})(m\phi_1+n\phi_2-\omega t)}\left|a_{m,n}\right\rangle + \Phi_{m,n}(t)e^{-(\frac{i}{2})(m\phi_1+n\phi_2-\omega t)}\left|b_{m,n}\right\rangle\right) \qquad (8)$$

Here $\psi_{m,n}(t)$ and $\Phi_{m,n}(t)$ are the probability amplitudes.

The complete basic system can be modeled by the nonlinear Schrodinger equations for wave function $\left|\Psi(t)\right\rangle$ and the following system of coupled nonlinear evolution equations for the probability amplitudes:

$$\frac{\partial\psi_{m,n}}{\partial t} = iG\psi_{m,n} + i\left(\zeta^{(a)}_{m,n}\psi_{m-1,n}e^{-i\varphi_1} + \zeta^{(a)}_{m+1,n}\psi_{m+1,n}e^{i\varphi_1} + \zeta^{(a)}_{m,n}\psi_{m,n-1}e^{-i\varphi_2} + \zeta^{(a)}_{m,n+1}\psi_{m,n+1}e^{i\varphi_2}\right) - ig\Phi_{m,n} - i\Delta\omega\left|\Phi_{m,n}\right|^2\psi_{m,n} \qquad (9)$$

$$\frac{\partial\Phi_{m,n}}{\partial t} = iG\Phi_{m,n} + i\left(\zeta^{(b)}_{m,n}\Phi_{m-1,n}e^{i\varphi_1} + \zeta^{(b)}_{m+1,n}\Phi_{m+1,n}e^{-i\varphi_1} + \zeta^{(b)}_{m,n}\Phi_{m,n-1}e^{i\varphi_2} + \zeta^{(b)}_{m,n+1}\Phi_{m,n+1}e^{-i\varphi_2}\right) - ig\psi_{m,n} - i\Delta\omega\left|\psi_{m,n}\right|^2\Phi_{m,n} \qquad (10)$$

Here G is the detuning parameter [8], $\varphi = \frac{kd}{2}$ is the phase shift, $g = -\mu E_0/2\hbar$ represents to the quantum dot field coupling factor and $\Delta\omega = 4\pi\mu_\alpha\mu_\beta M_{\alpha\beta}/\hbar V$ being the depolarization shift. $\zeta^{(a)}$ and $\zeta^{(b)}$ are the intersite coupling coefficients. Here equations (9) & (10) are used as the basis model for the analysis of Rabi oscillation solitons in the assembly of quantum dots.

3 ANALYTICAL CALCULATIONS

The above system can be normalized as:

$$\sum_{m=-N_1/2}^{N_1/2}\sum_{n=-N_2/2}^{N_2/2}\left(\left|\psi_{m,n}\right|^2 + \left|\Phi_{m,n}\right|^2\right) = 1 \qquad (11)$$

The energy of the system can be observed as:

$$\varepsilon = \left\langle\hat{H}\right\rangle = \frac{1}{2}\sum_{m=-N_1/2}^{N_1/2}\sum_{n=-N_2/2}^{N_2/2}\Big[-G\left(\left|\psi_{m,n}\right|^2 - \left|\Phi_{m,n}\right|^2\right) - \Phi^*_{m,n}\left(\zeta^{(b)}_{m,n}\Phi_{m-1,n}e^{i\varphi_1} + \zeta^{(b)}_{m+1,n}\Phi_{m+1,n}e^{-i\varphi_1} + \zeta^{(b)}_{m,n}\Phi_{m,n-1}e^{i\varphi_2} + \zeta^{(b)}_{m,n+1}\Phi_{m,n+1}e^{-i\varphi_2}\right) - \psi^*_{m,n}\left(\zeta^{(a)}_{m,n}\psi_{m-1,n}e^{-i\varphi_1} + \zeta^{(a)}_{m+1,n}\psi_{m+1,n}e^{i\varphi_1} + \zeta^{(a)}_{m,n}\psi_{m,n-1}e^{-i\varphi_2} + \zeta^{(a)}_{m,n+1}\psi_{m,n+1}e^{i\varphi_2}\right) + g\psi^*_{m,n}\Phi_{m,n} + \frac{1}{2}\Delta\omega\left|\psi_{m,n}\right|^2\left|\Phi_{m,n}\right|^2 + c.c.\Big] \qquad (12)$$

We assume the intersite coupling coefficients for ground and excited states to be equal i.e. $\zeta^{(a)} = \zeta^{(b)} = \zeta$, g=-1 and sign Δω=-1. All the signs indicate the attractive onsight linear interaction between the fields $\psi_{m,n}$ and $\Phi_{m,n}$.

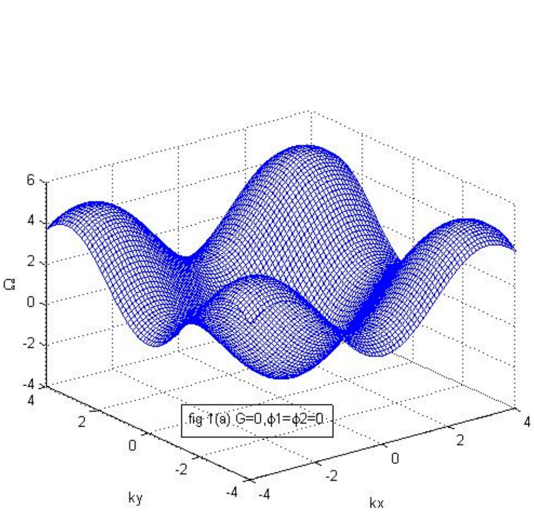
Let us generalize the 2-dimensional Rabi-wave model in the system of quantum dots assembly by considering Δω=0. By solving equations (9) & (10), we get the simplified version of the system as:

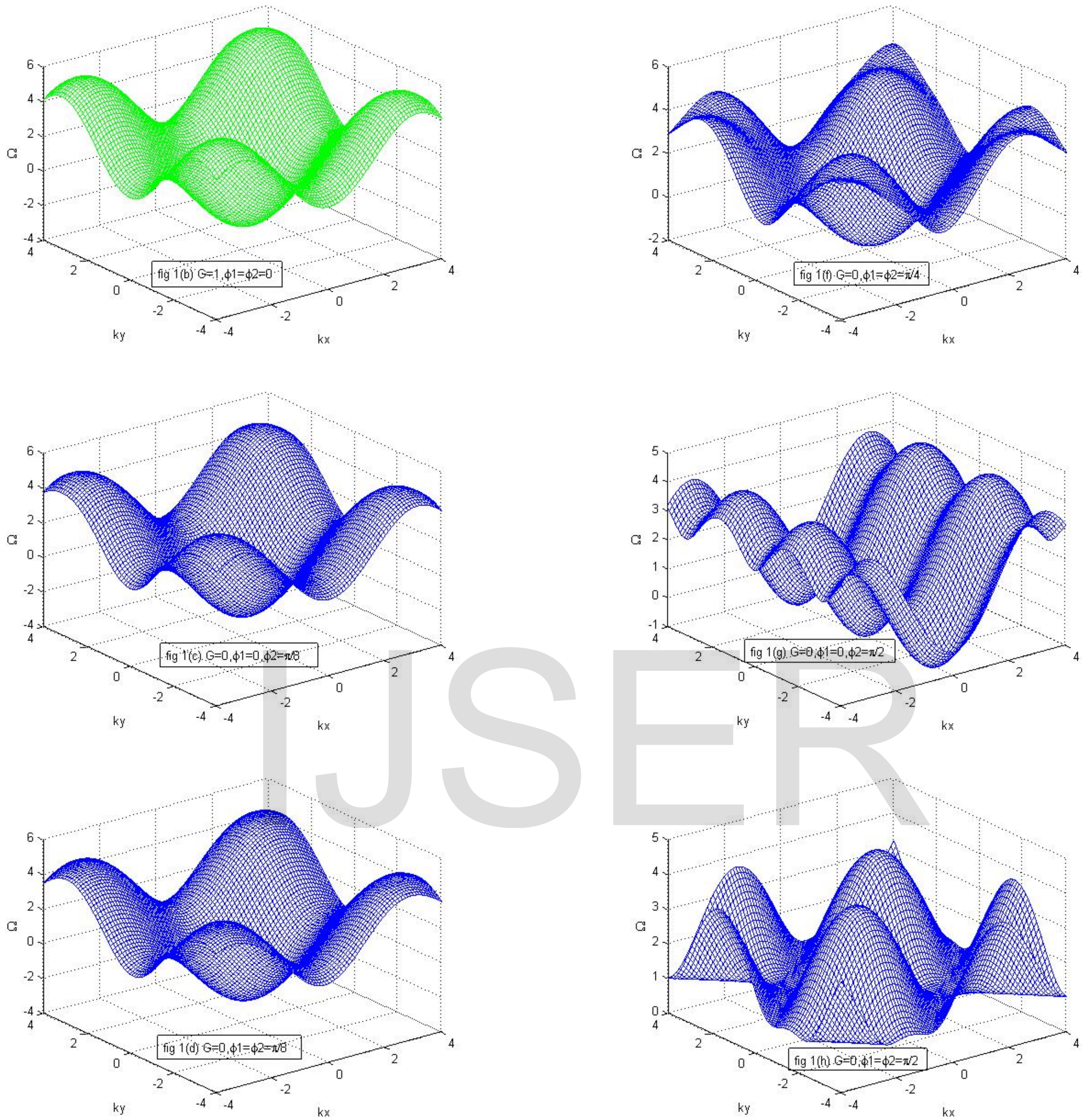
$$\left\{\psi_{m,n}, \Phi_{m,n}\right\} = \left\{P, Q\right\}\exp\left(i(k_x m + k_y n)\right)\exp(-i\Omega t) \qquad (13)$$

Here P & Q are unknown wave amplitudes, k_x & k_y are wavenumbers and Ω is an unknown frequency. By the analysis of equation (13), we get two branches of the dispersion relation

$$\Omega_{1,2}(k_x, k_y) = -2\zeta\left[\cos(k_x)\cos(\phi_1) + \cos(k_y)\cos(\phi_2)\right] \pm\sqrt{g^2 + \left\{G - 2\zeta[\sin(k_x)\sin(\phi_1) + \sin(k_y)\sin(\phi_2)]\right\}^2} \qquad (14)$$

Considering constants ζ & g fixed as ζ=1 & g=-1, we get the dispersion curves among Ω, k_x & k_y at different combinations of parameters G & ϕ, as shown in the following figure:





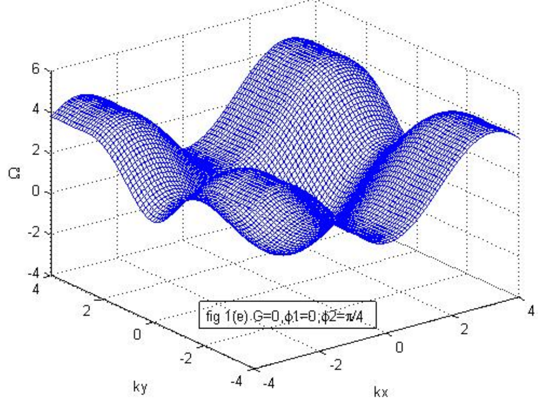


Figure 1. Dispersion curves among Ω, k_x and k_y at fixed values of coefficients ζ=1 & g=-1. Blue surfaces are drawn for G=0 and green surface is for G=1. These curves clearly show the existence of discrete solitons.

3.1 Rabi Waves

Let us substitute $\phi_1 = \phi_2 = \phi = 0$ in equation (14), we get a known dispersion relation for the system of linearly coupled 2-dimensional discrete nonlinear Schrodinger equations [6]. It shows two similar branches with a constant shift $\Delta\Omega = 2\sqrt{g^2 + G^2}$ which corresponds to Rabi waves. When $\phi_1 = \phi_2 \neq 0$ shift ΔΩ depends on wavenumbers k_x & k_y and become variable.

Moreover,

$$\Omega\left(k_x, k_y, \phi_1, \phi_2\right) = \Omega\left(-k_x, -k_y, -\phi_1, -\phi_2\right) \quad (15)$$

And $\Omega\left(k_x, k_y, \phi_1, \phi_2\right) \neq \Omega\left(-k_x, -k_y, \phi_1, \phi_2\right)$ (16)

From equation (15) we can see that inverting the propagation direction of the driving fields, the direction of the Rabi-wave propagation also get inverted.

4 Results and Discussion

By solving equations (9) & (10), we can get stationary discrete 2-dimensional fundamental soliton solutions as follows:

$$\left\{\psi_{m,n}, \Phi_{m,n}\right\} = e^{-i\Omega t}\left\{P_{m,n}, Q_{m,n}\right\} \quad (17)$$

Here $P_{m,n}$ & $Q_{m,n}$ are localized complex lattice fields vanishing at infinity. The stationary soliton solutions of equations (9) & (10) are numerically obtained with the help of nonlinear equation solver based on Powell method. The direct dynamical simulations are received by the Runge-Kutta procedure [10].

In this system the field emission is described by the field operator $\hat{M}$ with the help of Heisenberg representation [6] as:

$$\hat{E}(\bar{r}, t) = -\frac{1}{4\pi\varepsilon_0}\left(\nabla.\nabla - \frac{1}{c^2}\frac{\partial^2}{\partial t^2}\right)\int_V \frac{1}{\left|\bar{r}-\bar{r}'\right|}\hat{M}\left(t - \frac{\left|\bar{r}-\bar{r}'\right|}{c}\right)d^3 r' \quad (18)$$

Here, the operator $\hat{M}$ of the induced polarization explains the displacement current produced by the soliton profile and plays an important role in achieving the radiative properties of the quantum dots assembly.

The field produced by the solitons in the quantum dot assembly is small, linear approximation is used to determine the emission of such field and the total field produced by the complete system is achieved by superposition of partial fields emitted by different quantum dots independently.

Let us represent the far zone field in the spherical coordinates assuming origin at the central point of the cell keeping m=n=0, as $x = R\sin(\theta)\cos(\phi)$, $y = R\sin(\theta)\sin(\phi)$ & $z = R\cos(\theta)$. Here the higher order terms are omitted and longitudinal components of the electric field also vanish and we achieve the variable field as:

$$E_{Rad} = \lim_{R\to\infty} E = \frac{\mu e_\theta}{4\pi\varepsilon_0}\frac{\omega^2}{c^2}\frac{e^{-i\omega(t-R/c)}}{R}G\left(\theta, \varphi, \phi_1, \phi_2; t - R/c\right) + c.c. \quad (19)$$

With the radiation pattern as:

$$G\left(\theta, \varphi, \phi_1, \phi_2; t - R/c\right) = \sin\theta \sum_{m=-N_1/2}^{N_1/2}\sum_{n=-N_2/2}^{N_2/2}\psi_{m,n}(\tilde{t})\Phi^*_{m,n}(\tilde{t})$$

$$\times\exp\left\{im\left[\frac{\omega a}{c}\sin\theta\cos\varphi + 2\phi_1\right] + in\left[\frac{\omega a}{c}\sin\theta\sin\varphi + 2\phi_2\right]\right\} \quad (20)$$

Here $\tilde{t} = t - \frac{R}{c} + \frac{1}{c}\sin\theta\left[m\cos\varphi + n\sin\varphi\right]$ and R, θ, φ are the spherical coordinates. From equations (9), (10), (19) & (20) using different parameters as Ω=-10, ζ=0.15, G=1 and $\phi_1=\phi_2=\pi/4$, we get the radiation pattern as shown in figure-2. From these equations and figure-2 it is obvious that the radiation pattern is non-steady and depends on the spatio-temporal variable $\left(t - \frac{R}{c}\right)$.

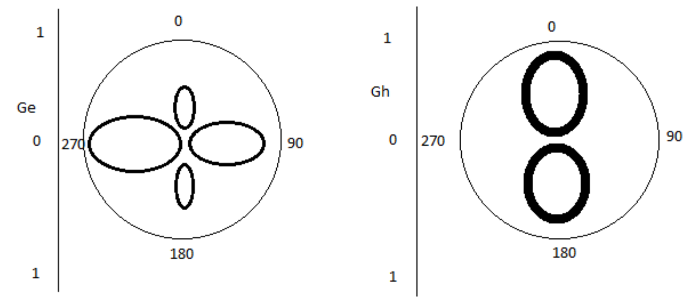


Figure 2. Radiation patterns observed in E and H planes by chosing the values of different parameters as Ω=-10, ζ=0.15, G=1 and $\phi_1=\phi_2=\pi/4$.

5 Conclusion

Considering 2-dimensional assembly of quantum dots and using two discrete nonlinear Schrodinger equations, we have thus modeled the system. Local field effects were taken into account, which introduce the nonlinearity of the electron hole motion inside each quantum dot. Numerical calculations were performed and analytical computations were made in MATLAB to achieve different dispersion curves at different values of parameters showing the presence of discrete solitons.

Interaction of solitons with the assembly of quantum dots in the form of Rabi oscillations has been observed. Thus, we achieved radiative properties of solitons and the radiation pattern has been shown as figure-2. Our study has also shown a wide future scope of research in the field of quantum dots.

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